PROBLEMS:

1) Compute the first 3 steps of the initial value problem $\frac{dy}{dx} = \frac{x-y}{2}$, $y(0) = 1.0$ by Taylor series method and next step by Milne's method with step length $h = 0.1$.

Solution :-

Here $x_0 = 0$, $y_0 = 1$, $x_1 = 0.1$, $x_2 = 0.2$, $x_3 = 0.3$, $x_4 = 0.4$, $h = 0.1$.

$y' = \frac{1}{2}(x-y)$, $y_0' = \frac{1}{2}(x_0 - y_0) = \frac{1}{2}(0-1) = -\frac{1}{2}$.

$y'' = \frac{1}{2}[1 - y']$, $y_0'' = \frac{1}{2}[1 - y_0'] = \frac{1}{2}[1 + \frac{1}{2}] = 3/4$

$y''' = \frac{1}{2}[-y'']$, $y_0''' = \frac{1}{2}[-y_0''] = \frac{1}{2}[-\frac{3}{4}] = \frac{-3}{8}$.

$$y_1 = y_0 + h y_0' + \frac{h^2}{2!} y_0'' + \frac{h^3}{3!} y_0''' + \cdots$$

$$= 1 + (0.1)(-\tfrac{1}{2}) + \frac{(0.1)^2}{2}(3/4) + \frac{(0.1)^3}{6}(-\tfrac{3}{8}) + \cdots$$

$$= 1 - \frac{0.1}{2} + \frac{0.03}{8} - 0.0000625$$

$$= 1 - 0.05 + 0.00375 - 0.0000625$$

$y(0.1) = 0.95369$

$$\boxed{y(0.1) = 0.9537}$$

To find $y(0.2)$

$$y_1' = \frac{1}{2}(x_1 - y_1) = \frac{1}{2}(0.1 - 0.9537) = -0.4269$$

$$y_1'' = \frac{1}{2}[1 - y_1'] = \frac{1}{2}[1 + 0.4269] = 0.71345$$

$$y_1''' = \frac{1}{2}[-y_1''] = -\frac{1}{2}[0.71345] = -0.3567$$

$$y_2 = y_1 + h y_1' + \frac{h^2}{2!} y_1'' + \frac{h^3}{3!} y_1''' + \cdots$$

$$= 0.9537 + (0.1)(-0.4269) + \frac{(0.2)^2}{2}(0.71345) + \frac{(0.1)^3}{6}(-0.3567)$$

$$= 0.9537 - 0.04269 + 0.00357 - 0.00006 = 0.9145.$$

$$\therefore \boxed{y_2 = y(0.2) = 0.9145}$$

To find $y(0.3)$

$$y_2' = \frac{1}{2}(x_2 - y_2) = \frac{1}{2}(0.2 - 0.9145) = -0.3573$$

$$y_2'' = \frac{1}{2}(1 - y_2') = \frac{1}{2}(1 + 0.3573) = 0.6787$$

$$y_2''' = \frac{1}{2}[-y_2''] = -\frac{1}{2}(0.6787) = -0.3394$$

$$y_3 = y_2 + h y_2' + \frac{h^2}{2!} y_2'' + \frac{h^3}{3!} y_2''' + \cdots$$

$$= 0.9145 + (0.1)(-0.3573) + \frac{(0.1)^2}{2}(0.6787) + \frac{(0.1)^3}{6}(-0.3394)$$

$$= 0.9145 - 0.03573 + 0.00339 - 0.00006 = 0.8821$$

$$\therefore \boxed{y_3 = y(0.3) = 0.8821}$$

$$y_3' = \frac{1}{2}[x_3 - y_3] = \frac{1}{2}[0.3 - 0.8821] = -0.2911.$$

To find $y(0.4)$ using Milne's method:

By Milne's predictor formula

$$y_{4,P} = y_0 + \frac{4h}{3}[2y_1' - y_2' + 2y_3']$$

$$= 1 + \frac{4(0.1)}{3}[2(-0.4269) - (-0.3578) + 2(-0.2911)]$$

$$= 1 + \frac{0.4}{3}[-0.8538 + 0.3578 - 0.5822]$$

$$= 0.8562.$$

$$\boxed{y_{4,P} = 0.8562}.$$

$$y_4' = \frac{1}{2}[x_4 - y_4] = \frac{1}{2}[0.4 - 0.8562] = -0.2281$$

Using Milne's corrector formula

$$y_{4,C} = y_2 + \frac{h}{3}[y_2' + 4y_3' + y_4']$$

$$= 0.9145 + \frac{0.1}{3}[-0.3573 + 4(-0.2911) - 0.2281]$$

$$= 0.9145 + \frac{0.1}{3}[-0.3573 - 1.1644 - 0.2281]$$

$$= 0.8562$$

$$\therefore \boxed{y(0.4) = 0.8562}$$

(OR)

$$y(x) = y_0 + \frac{(x-x_0)}{1!} y_0' + \frac{(x-x_0)^2}{2!} y_0'' + \frac{(x-x_0)^3}{3!} y_0''' + \cdots$$

Here $x_0 = 0$, $y_0 = 1$.

$$y(x) = y_0 + \frac{x}{1!} y_0' + \frac{x^2}{2!} y_0'' + \frac{x^3}{3!} y_0''' + \cdots$$

$$y(x) = 1 + \frac{x}{1!} \cdot \left(-\frac{1}{2}\right) + \frac{x^2}{2!} (3/4) + \frac{x^3}{3!} \left(\frac{-3}{8}\right) + \cdots$$

<u>To find $y(0.1)$</u>

$$y_1 = y(0.1) = 1 + \frac{0.1}{1!}\left(-\frac{1}{2}\right) + \frac{(0.1)^2}{2}(3/4) + \frac{(0.1)^3}{6}(-3/8) + \cdots$$

$$\boxed{y(0.1) = 0.9537}$$

$$y_2 = y(0.2) = 1 + \frac{0.2}{1}\left(-\frac{1}{2}\right) + \frac{(0.2)^2}{2}(3/4) + \frac{(0.2)^3}{6}(-3/8) + \cdots$$

$$\boxed{y_2 = y(0.2) = 0.912}$$

$$y_3 = y(0.3) = 1 + \frac{0.3}{1}\left(\frac{-1}{2}\right) + \frac{(0.3)^2}{2}(3/4) + \frac{(0.3)^3}{6}\left(\frac{-3}{8}\right) + \cdots$$

$$\boxed{y_3 = y(0.3) = 0.882}$$

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2. Given $y' = x^2 + y$, $y(0) = 1$, find $y(0.1)$ by Taylor Series method, $y(0.2)$ by Modified Euler's method $y(0.3)$ by Runge-Kutta method and $y(0.4)$ by Milne's method.

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Ans:

$y_1 = y(0.1) = 1.1055$

$y_2 = y(0.2) = 1.2241$

$y_3 = y(0.3) = 1.3594$

$y_{4,P} = 1.5144$

$y_{4,C} = 1.5148$.

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